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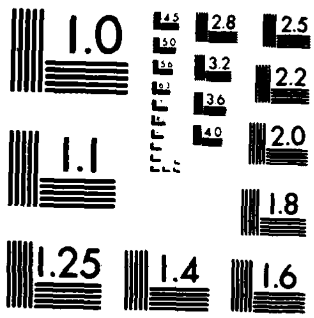
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RESONANCE ZONES IN TWO-PARAMETER FAMILIES OF CIRCLE HOMEOMORPHISMS

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ABSTRACT

We consider a two-parameter family of diffeomorphisms of the circle where one of the parameters controls the amount of rigid rotation while the second controls the nonlinearity. In particular, we show that the regions in the parameter plane for which the map has a periodic orbit of a particular rotation number (resonance zones) increase in size linearly as the second parameter is increased from zero. This is a discretization of the phenomenon known as "phase locking" for ordinary differential equations. Using this we obtain some results on the smoothness of the curves between the resonance zones.

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SIGNIFICANCE AND EXPLANATION

Maps of the circle, when viewed as discrete dynamical systems, have been observed to display many of the qualitatively complicated dynamical properties of experimental situations. One aspect of this qualitative behavior is the interplay between the existence and nonexistence of resonant or periodic behavior as parameters of the system are varied. For smooth diffeomorphisms of the circle it turns out that either every orbit tends to a periodic orbit of some fixed period, or every orbit is dense.

In this report we study maps near a rigid rotation of the circle and show that if a "typical" nonlinearity is added, one expects the regions of resonance to grow at a linear rate as the nonlinearity is increased. Using this we study the properties of the regions of 'non-resonance' in the parameter plane which exist between the resonance zones.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

RESONANCE ZONES IN TWO-PARAMETER FAMILIES OF CIRCLE HOMEOMORPHISMS

Glen Richard Hall

0) INTRODUCTION: Let $T = R/Z$ be the circle with unit circumference and consider the two parameter family of maps from T onto T given by

$$\theta \to \langle\theta + \phi + \alpha\gamma(\theta)\rangle \tag{1}$$

where $\langle\cdot\rangle$ denotes fractional part, ϕ and α are parameters and γ is a smooth function, periodic with period one. When $\alpha = 0$ this map is merely 'rigid rotation' by ϕ hence it will have periodic orbits if and only if ϕ is rational. When $\alpha > 0$ the set of parameter values where a periodic orbit exists of a particular period and rotation number open into a region in the (ϕ,α) plane (see Brunovsky [2]). In this paper we study the rate at which these resonance zones open near $\alpha = 0$. This can be considered a discretization of the phenomenon of 'phase locking' in O.D.E.'s which has been extensively studied (see, for example, Loud [7], Bushard [3,4]).

When α is small the resonance zones corresponding to periodic orbits of different period or rotation number will remain disjoint. Between these zones there will be arcs in the parameter plane where the above map has no periodic orbits and all orbits are dense. Herman [5] has shown that for $\alpha_0 > 0$, α_0 small the set of ϕ such that at parameter (ϕ,α_0) the map (1) has no periodic orbits will have positive measure. Herman [5] also showed that this measure tends to one as α tends to zero. Arnol'd [1] showed that certain of these arcs of non-resonance will be smooth depending on a number theoretic condition on the rotation number. In the final section of this paper we point out that although all the non-resonance arcs will have a first derivative at $\alpha = 0$, for certain rotation numbers (depending on γ) they will not have

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second derivatives at $\alpha = 0$. The precise nature of the non-resonance arcs for arbitrary irrational rotation number remains open.

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1) DEFINITIONS AND NOTATIONS: We let $\gamma : \mathbb{R} \to \mathbb{R}$ be a function satisfying

$$(1) \quad \gamma \in C^1 \text{ and } \left|\frac{d\gamma}{d\theta}\right| < 1 ,$$

$$(2) \quad \forall\, \theta \in \mathbb{R},\ \gamma(\theta + 1) = \gamma(\theta) ,$$

$$(3) \quad \int_0^1 \gamma(\theta)d\theta = 0 .$$

Given such a γ we may define a two parameter family of maps $f : \mathbb{R}^3 \to \mathbb{R}$ by

$$f : (\theta,\phi,\alpha) \to \theta + \phi + \alpha\gamma(\theta) .$$

The parameter ϕ controls the rotation or "twist" while α controls the nonlinearity of f. Note that for fixed (ϕ,α) we have

$$\forall\, \theta \in \mathbb{R},\ f(\theta + 1,\phi,\alpha) = f(\theta,\phi,\alpha) + 1$$

hence f is the lift of a two-parameter family of degree one map of the circle. By condition (1) on γ, f is a homeomorphism when $\alpha \in [0,1)$.

Notation: For fixed (ϕ,α) we let $f^n(\theta,\phi,\alpha)$ denote the n^{th} iterate of $f(\cdot,\phi,\alpha)$, i.e., $f^n(\theta,\phi,\alpha) = f(f^{n-1}(\theta,\phi,\alpha),\phi,\alpha)$.

Definition: For $\phi \in \mathbb{R}$, $\alpha \in [0,1)$ the rotation number of $f(\cdot,\phi,\alpha)$ is defined to be

$$\rho(\phi,\alpha) = \lim_{n\to\infty} \frac{f^n(\theta,\phi,\alpha) - \theta}{n}$$

We will use the following facts about the rotation number due to Poincare.

Theorem A: For $\phi \in \mathbb{R}$, $\alpha \in [0,1)$

1) $\rho(\phi,\alpha)$ exists and is independent of the θ in the definition,

2) $\rho(\phi,\alpha)$ is continuous in (ϕ,α) and increasing in ϕ,

3) if $\rho(\phi,\alpha) = p/q \in \mathbb{Q}$ then there exists $\theta \in [0,1)$ such that $f^q(\theta,\phi,\alpha) = \theta + p$.

Proof: See Herman [6]. //

Remark: Since $f(\cdot,\phi,\alpha)$ is the lift of a homeomorphism of a circle when $\alpha \in [0,1)$, we may reinterpret part (3) of Theorem A as saying that this circle map has a periodic orbit with period q and rotation number p/q.

Definition: For $\beta \in \mathbb{R}$ we let

$$A_\beta = \{(\phi,\alpha) : \phi \in \mathbb{R},\ \alpha \in [0,1),\ \rho(\phi,\alpha) = \beta\} .$$

Remark: When $\beta = p/q$ is rational then the set $A_{p/q}$ is called the p/q resonance horn or the p/q Arnold tongue.

Notation: We say $\gamma \in C^{r+\phi}$ for $r > 1$ an integer and $\varepsilon \in (0,1]$ if $\gamma \in C^r$ and there exists a constant $c > 0$ such that

$$\forall\ \theta_1,\theta_2 \in \mathbb{R},\ \left|\frac{d^r\gamma}{d\theta^r}(\theta_1) - \frac{d^r\gamma}{d\theta^r}(\theta_2)\right| < C|\theta_1 - \theta_2|^\varepsilon .$$

If $\gamma \in C^{r+\varepsilon}$ and is given by the Fourier series $\gamma(\theta) = \sum_{n=-\infty}^{\infty} a_n e^{in2\pi\theta}$ then $|a_n| = \theta(|n|^{-r-\varepsilon})$ (see [1]).

2) Resonance horns: For γ as in section (1) and rational p/q in lowest terms, we wish to consider the set $A_{p/q}$ for α near zero.

Theorem 1 (See Herman [6]): There exists Lipschitz functions $\phi_1,\phi_2 : [0,1) \to \mathbb{R}$

1) $\forall\ \alpha \in [0,1),\ \phi_1(\alpha) < \phi_2(\alpha)$,

2) $\phi_1(0) = p/q = \phi_2(0)$,

(3) $(\phi,\alpha) \in A_{p/q}$ if and only if $\phi_1(\alpha) < \phi < \phi_2(\alpha)$.

Moreover, the Lipschitz constant of ϕ_1,ϕ_2 is independent of p/q (i.e. it depends only on γ).

We will include a proof of Theorem 1 since it allows us to set up notation for

Theorem 2: For each integer $r > 1$ and $\delta \in (0,1]$ and constant $c > 0$ there exists $\gamma \in C^r$ such that for any rational k/s in lowest terms there exists $\varepsilon_{k/s} > 0$ such that for $\alpha \in [0,\varepsilon_{k/s}]$ we have

$$\min\{\phi : (\phi,\alpha) \in A_{k/s}\} <$$

$$< \alpha(k/s - c/s^{r+1+\delta}) < \alpha(k/s + c/s^{r+1+\delta}) < \max\{\phi : (\phi,\alpha) \in A_{k/s}\} .$$

Remark: Theorem (2) says that each horn "opens" about the vertical ray eminating from its tip (see Figure 1). Since the horns must

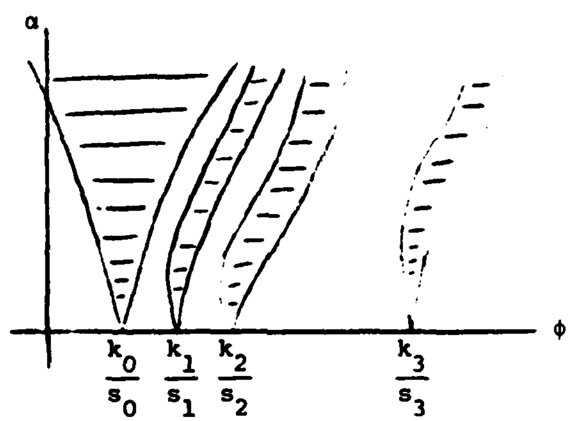


Figure 1

remain disjoint, the $\varepsilon_{k/s} \to 0$ as $s \to \infty$.

Proof of Theorem 1: Since, by Theorem A, the function $\rho(\phi,\alpha)$ is increasing in ϕ we may define functions $\phi_1,\phi_2 : [0,1) \to R$ satisfying (1,2,3) of Theorem 1. It remains to show ϕ_1,ϕ_2 are Lipschitz. For each $\theta_0 \in [0,1)$ we note that

$$f^q(\theta_0,\phi_1(\alpha),\alpha) < \theta_0 + p$$

$$f^q(\theta_0,\phi_2(\alpha),\alpha) > \theta_0 + p ,$$

and since $\frac{\partial f^q}{\partial\phi}(\theta,\phi,\alpha) > 1$ for all (θ,ϕ,α) we may use the Implicit Function Theorem to obtain a curve $\phi_{\theta_0} : [0,1) \to R$ such that $f^q(\theta_0,\phi,\alpha) = \theta_0 + p$ if and only if $\phi = \phi_{\theta_0}(\alpha)$. Moreover $\phi_{\theta_0}(0) = p/q$ and

$$\frac{d\phi_{\theta_0}}{d\alpha}(\alpha) = -\frac{\partial f^q}{\partial\alpha}(\theta_0,\phi_{\theta_0}(\alpha),\alpha) \Big/ \frac{\partial f^q}{\partial\phi}(\theta_0,\phi_{\theta_0}(\alpha),\alpha) ,$$

and using the chain rule we obtain

$$\left|\frac{d\phi_{\theta_0}}{d\alpha}(\alpha)\right| < \sup_{\theta\in[0,1)} |\gamma(\theta)| .$$

By Theorem A part (3) we see that

$$\phi_1(\alpha) = \sup_{\theta_0\in[0,1)} \phi_{\theta_0}(\alpha) \quad \text{and} \quad \phi_2(\alpha) = \sup_{\theta_0\in(0,1)} \phi_{\theta_0}(\alpha) .$$

Hence ϕ_1,ϕ_2 are Lipschitz with constant independent of p/q. //

Remark: In fact, the boundaries of $A_{p/q}$ are characterized by the existence of a node, i.e. a point $\theta \in [0,1)$ such that $f^q(\theta,\phi,\alpha) = \theta + p$ and $\frac{df^q}{d\theta}(\theta,\phi,\alpha) = 1$. If $\gamma \in C^2$ then we may apply the Implicit Function Theorem to these equations to obtain the boundary curves of $A_{p/q}$. Hence, generically, these curves will be piecewise as smooth as γ.

Proof of Theorem 2: Noting that for any rational, say p/q,

$$f^q(\theta,\phi,\alpha) = \theta + q\phi + \alpha\sum_{j=0}^{q-1}\gamma(\theta + j\phi) + \alpha h(\theta,\phi,\alpha)$$

where h is C^1 and $h \to 0$ as $\alpha \to 0$, we see that, using the notation above,

$$\frac{d\phi_{\theta_0}}{d\alpha}(0) = -\sum_{j=0}^{q-1}\gamma(\theta_0 + j\,p/q)/q .$$

Hence if $\gamma(\theta) = \sum_{n=-\infty}^{\infty} a_n e^{in2\pi\theta}$, we have

$$(*) \qquad \frac{d\phi_{\theta_0}}{d\alpha}(0) = -\sum_{n=-\infty}^{\infty} a_{nq} e^{inq2\pi\theta_0} .$$

For $c > 0$ fixed, take

$$\gamma(\theta) = \sum_{n=1}^{\infty} \frac{3c}{n^{r+\delta+1}} (e^{in2\pi\theta} + e^{-in2\pi\theta}) .$$

Then $\gamma \in C^r$ (see [1]) and

$$\frac{d\phi_{\theta_0}}{d\alpha}(0) = \frac{6c}{q^{r+\delta+1}} \cos(2\pi q^{r+\delta+1}\theta_0) + \sum_{|n|\geq 2} \frac{3c}{(|n|q)^{r+\delta+1}} e^{inq2\pi\theta_0} .$$

But

$$\left| \sum_{|n|\geq 2} \frac{1}{(|n|q)^{r+\delta+1}} e^{inq2\pi\theta} \right| < \frac{1}{q^{r+\delta+1}} \sum_{|n|\geq 2} \frac{1}{|n|^2} < 1.6 \frac{1}{q^{r+\delta+1}}$$

Hence

$$\sup_{\theta_0 \in [0,1)} \frac{d\phi_{\theta_0}}{d\alpha}(0) > \frac{1.2c}{q^{r+\delta+1}} > \frac{-1.2c}{q^{r+\delta+1}} > \inf_{\theta_0 \in [0,1)} \frac{d\phi_{\theta_0}}{d\alpha}(0)$$

and since p/q was an arbitrary rational, the proof of the theorem is complete. //

As an easy corollary we see that, in a strong sense, it is typical for the resonance horns to open at a positive rate.

<u>Corollary 1</u>: If $\gamma(\theta) = \sum_{n=-\infty}^{\infty} a_n e^{in2\pi\theta}$ is as above then for each rational k/s there exists $\varepsilon_{k/s} > 0$ such that if $\alpha \in [0, \varepsilon_{k/s}]$

$$(**) \qquad \text{length}\{\phi : (\phi,\alpha) \in A_{k/s}\} > \alpha\Big(\sum_{n=-\infty}^{\infty} |a_{nq}|^2\Big)^{1/2}/2 .$$

and if the right hand side of (**) is non-zero then

$$\min\{\phi : (\phi,\alpha) \in A_{k/s}\} < k/s < \max\{\phi : (\phi,\alpha) \in A_{k/s}\} .$$

<u>Proof</u>: Fixing a rational p/q, we see from (*) above that the L^2 norm $(d\phi_{\theta_0}/d\alpha)(0)$ as a function of θ_0 is $\left(\sum_{n=-\infty}^{\infty} |a_{nq}|^2\right)^{1/2}$. But $\int_0^1 (d\phi_{\theta_0}/d\alpha)(0)d\theta_0 = a_0 = 0$ so if this norm is positive then $(d\phi_{\theta_0}/d\alpha)(0)$ must assume both negative and positive values and the proof is complete. //

<u>Remark 1</u>: If we take $\gamma(\theta) = \sin(2\pi\theta)/2\pi$ then all the higher Fourier coefficients are zero. In fact, the edges of the resonance horns will have high order contact at $\alpha = 0$ depending on the denominator of the rotation number (see Arnol'd [1]).

2) Let $A_\alpha = \{\phi : \rho(\phi,\alpha) \notin \mathbb{Q}\}$ and let λ be Lebesgue measure on $\mathbb{R}$. As noted in the introduction, Herman [5] has shown that for any bounded interval $I \subseteq \mathbb{R}$, and $0 < \alpha < 1$, $\lambda(A_\alpha \cap I) > 0$ whenever $A_\alpha \cap I \neq \emptyset$ and $\lambda(A_\alpha \cap I) \to \lambda(I)$ as $\alpha \to 0$. An immediate consequence of Corollary 1 is

<u>Corollary 2</u>: Let $\gamma(\theta) = \sum_{n=-\infty}^{\infty} a_n e^{in\theta}$ be as above with $a_n \neq 0$ for infinitely many n. Then for any bounded interval $I \subseteq \mathbb{R}$ there exist constants $\varepsilon_I, c_I > 0$ such that

$$\lambda(A_\alpha \cap I) < \lambda(I)(1 - c_I\alpha)$$

whenever $\alpha \in [0,\varepsilon_I]$.

3) <u>NON-RESONANCE</u>: Fix γ as in section (1). For a given irrational ζ there exists a Lipschitz curve $\phi_\zeta : [0,1) \to \mathbb{R}$ such that $\rho(\phi,\alpha) = \zeta$ if and only if $\phi = \phi_\zeta(\alpha)$ (see Herman [6]).

<u>Theorem 3</u>: For any irrational ζ, the derivative of $\phi_\zeta(\alpha)$ exists when $\alpha = 0$ and equals 0.

<u>Proof</u>: Recall that since $\rho(\phi_\zeta(\alpha),\alpha) = \zeta$ there is a unique $f(\cdot,\phi_\zeta(\alpha),\alpha)$ invariant probability measure μ_α and

$$\zeta = \rho(\phi_\zeta(\alpha),\alpha) =$$

$$= \lim_{n\to\infty} \frac{f^n(\theta,\phi_\zeta(\alpha),\alpha) - \theta}{n} = \lim_{n\to\infty} \frac{1}{n} \sum_{j=0}^{n-1} (f^{j+1}(\theta,\phi_\zeta(\alpha),\alpha) - f^j(\theta,\phi_\zeta(\alpha),\alpha))$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{j=0}^{n-1} (f(\cdot,\phi_\zeta(\alpha),\alpha) - \text{identity})(f^j(\theta_1,\phi_\zeta(\alpha),\alpha)$$

$$= \int_0^1 (f(\theta,\phi_\zeta(\alpha),\alpha) - \theta)d\mu_\alpha(\theta)$$

$$= \phi_\zeta(\alpha) + \alpha \int_0^1 \gamma(\theta)d\mu_\alpha(\theta) .$$

(see Herman [6] for details). But noting that $\phi_\zeta(0) = \zeta$ we see that

$$\frac{1}{\alpha} (\phi_\zeta(\alpha) - \phi_\zeta(0)) = \int_0^1 \gamma(\theta)d\mu_\alpha(\theta)$$

and $\int_0^1 \gamma(\theta)d\mu_\alpha(\theta) \to \int_0^1 \gamma(\theta)d\theta = 0$ as $\alpha \to 0$ since $d\mu_0 = d\theta$. So the derivative of $\phi_\zeta(\alpha)$ exists at $\alpha = 0$ and equals zero. //

If, for example, γ is analytic and ζ is sufficiently poorly approximable by rationals then the curve ϕ_ζ will also be analytic (see Arnold [1]). However, this will not hold for all irrationals.

<u>Theorem 4</u>: For $\gamma(\theta) = \sum_{n=-\infty}^{\infty} a_n e^{in2\pi\theta}$ with infinitely many $a_n \neq 0$ there exist ζ irrational such that $\phi_\zeta(\alpha)$ does not have a second derivative at $\alpha = 0$, (i.e. $\lim_{\alpha\to 0} \frac{\phi_\zeta(\alpha) - \phi_\zeta(0)}{\alpha^2}$ does not exist).

<u>Proof</u>: Fix a rational p_1/q_1 in lowest terms such that $a_{q_1} \neq 0$. Let $c_1 < 0$ be greater than the slope of the left edge of A_{p_1/q_1}. Then for $\delta_1 = c_1/2 > 0$ there exists $\varepsilon_1 > 0$ such that if ζ is irrational and $0 < p_1/q_1 - \zeta < \varepsilon_1$ then there exists $0 < \alpha < \delta_1$ such that

$$|\phi_\zeta(\alpha) - \zeta| > |c_1|\alpha/2 \quad \text{so} \quad \frac{|\phi_\zeta(\alpha) - \zeta|}{\alpha^2} > 1 .$$

We may proceed by induction to produce $\frac{p_2}{q_2}, \frac{p_3}{q_3}, \ldots$ with $a_{q_j} \neq 0$ and the associated c_j, ε_j and $\delta_j = \frac{c_j}{2}$ such that for irrational ζ with $|\frac{p_j}{q_j} - \zeta| < \varepsilon_j$ there exists α with $0 < \alpha < \delta_j$

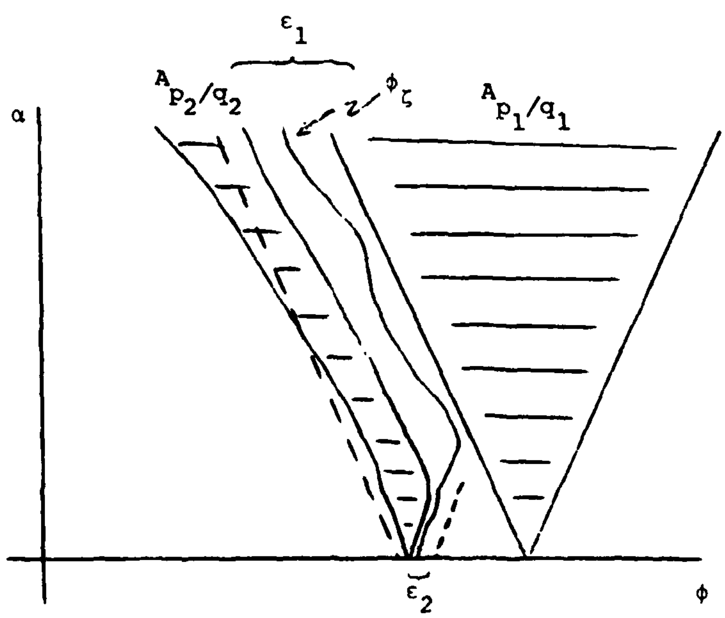
$$|\phi_\zeta(\alpha) - \zeta|/\alpha^2 > |c_j|/2\delta_j > 1 ,$$

$$|\frac{p_j}{q_j} - \frac{p_{j+1}}{q_{j+1}}| < \varepsilon_j ,$$

$$\text{and} \quad \frac{p_{2(j-1)}}{q_{2(j-1)}} < \frac{p_{2j}}{q_{2j}} < \frac{p_{2j+1}}{q_{2j+1}} < \frac{p_{2j-1}}{q_{2j-1}} .$$

Moreover, we may take $\varepsilon_j \to 0$ so fast that $\lim_{j\to\infty} \frac{p_j}{q_j} = \zeta_0$ is irrational. Then ζ_0 is the required irrational (see Figure 2). //

<u>Remark</u>: We conjecture that $\phi_\zeta(\alpha)$ is C^1 on $[0,1)$ for all irrational ζ. It is possible that $\phi_\zeta(\alpha)$ is even smoother on $(0,1)$ for arbitrary irrationals ζ.



<u>Figure 2</u>

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